

Exact Structures on Monomorphism Categories

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Monomorphism categories appear naturally in representation theory. For example, modules over truncated polynomial rings can be considered as vector spaces together with nilpotent operators, and their monomorphisms are related to invariant subspaces. These categories have been studied using triangular matrix rings, Auslander–Reiten theory, submodule categories, and quotient functors. Such studies show that monomorphism categories provide an interesting connection between module theory and homological algebra. Let Λ be an Artin algebra and let $\mathcal{X}$ be a resolving and contravariantly finite subcategory of $\text{mod-}\Lambda$. In this talk, we study the relative monomorphism category $\mathcal{S}_{\mathcal{X}}(\Lambda)$. Besides the canonical exact structure, we consider componentwise split and strongly componentwise split exact structures. We describe the projective and injective objects for these structures. We also study a functor from $\mathcal{S}_{\mathcal{X}}(\Lambda)$ to the category of finitely presented functors over the stable category $\underline{\mathcal{X}}$. Under a suitable exact structure, this functor is exact, full, dense, and objective. As applications, we obtain some equivalences involving stable, derived, and singularity categories. The results show how different exact structures can give different homological information from the same monomorphism category.